

Counting

Two most important topics in this module

- Addition rule of counting
- Multiplication rule of counting

Addition rule of counting

To count...

People in class: count men, count women, add together.

Fruit in bowl: count apples, count bananas, count oranges, ..., add together.

Size $|A|$ where $A = B \cup C$ and $B \cap C = \emptyset$: $|B| + |C|$.

Multiplication rule of counting

If

- There are m ways to do Step 1, and
- For each of those, there are n ways to do Step 2,

Then, there are mn ways to do both Step 1 and Step 2.

Multiplication rule of counting

Ex: 3 running for president, 2 for VP, 4 for treasure.

- There are 3 ways the president race could turn out.
- For each there are 2 ways VP race could turn out.
- So, there are 6 ways the president/VP pair could turn out.
- For each of 6 ways president/VP pair could turn out, there are 4 ways the treasurer could turn out.
- So, 24 ways president/VP/treas could turn out.

Multiplication rule of counting

Ex: 3 running for president, 2 for VP, 4 for treasurer.

Let A/B/C be running for president, D/E for VP, and F/G/H/I for treasurer.

Let (P,V,T) be how we write the result of the election.

(A, D, F)	(A, D, G)	(A, D, H)	(A, D, I)
(A, E, F)	(A, E, G)	(A, E, H)	(A, E, I)
(B, D, F)	(B, D, G)	(B, D, H)	(B, D, I)
(B, E, F)	(B, E, G)	(B, E, H)	(B, E, I)
(C, D, F)	(C, D, G)	(C, D, H)	(C, D, I)
(C, E, F)	(C, E, G)	(C, E, H)	(C, E, I)

Multiplication rule of counting

If you're trying to count how many objects meet some criteria, write a sequence of steps that specify an object of interest.

Ex:

1. Specify who wins president.
2. Specify who wins vice-president.
3. Specify who wins treasurer.

Multiplication rule of counting

If

- every object you wish to count has exactly one sequence of responses, and
- no object you DO NOT want to count can be specified,

then multiply the number of candidates for each step.

$$3 \times 2 \times 4 = 24$$

Multiplication rule examples

How many 4-digit numbers are there (1000-9999)?

Step 1: Select thousands digit 1-9. (9 candidates)

Step 2: Select hundreds digit 0-9. (10 candidates)

Step 3: Select tens digit 0-9. (10 candidates)

Step 4: Select ones digit 0-9. (10 candidates)

Steps can't create any bad numbers, good numbers have only one sequence of answers.

$$9 \times 10 \times 10 \times 10 = 9000$$

Multiplication rule examples

How many odd 4-digit numbers are there?

Step 1: Select thousands digit 1-9. (9 candidates)

Step 2: Select hundreds digit 0-9. (10 candidates)

Step 3: Select tens digit 0-9. (10 candidates)

Step 4: Select ones digit 1,3,5,7,9. (5 candidates)

Steps can't create any bad numbers, good numbers have only one sequence of answers.

$$9 \times 10 \times 10 \times 5 = 4500$$