Quiz guide solutions

Construct a simple relation on {1, 2, 3} that is reflexive but not symmetric. Give a counterexample for symmetry in the form of a sentence.

Reflexive: $1R1$, $2R2$, $3R3$ must all be there.

Symmetric: $xRy \to yRx$

Not symmetric: $xRy \land {\sim}yRx$

One answer: $R = \{(1,1),(2,2),(3,3),(1,2)\}$

Construct a simple relation on {1, 2, 3} that is reflexive but not symmetric. Give a counterexample for symmetry in the form of a sentence.

One answer: $R = \{(1,1),(2,2),(3,3),(1,2)\}$

R is not symmetric because $1R2$ *but not* $2R1$.

Construct a simple relation on {1, 2, 3} that is symmetric but not transitive. Give a counterexample for transitivity in the form of a sentence.

A relation is symmetric unless there is a counterexample. So avoid creating a counterexample.

Transitive: $xRy \wedge yRz \rightarrow xRz$

Not transitive: $xRy \wedge yRz \wedge {\sim}xRz$

$R = \{(1,2),(2,3)\}$ is not transitive. (Also not symmetric.)

Construct a simple relation on {1, 2, 3} that is symmetric but not transitive. Give a counterexample for transitivity in the form of a sentence.

One answer: $R = \{(1,2),(2,3),(2,1),(3,2)\}$

R is not transitive because $1R2$ *and* $2R3$ *but not* $1R3$.

Note: $R = \{(1,2),(2,1)\}$ is a smaller relation that is symmetric but not transitive.

Let $R = \{((a,b),(c,d)) \,:\, a+b=c+d\}$ *be a relation on* $\mathbb{Z}\times\mathbb{Z}$*. What is in the equivalence class* $[(2,3)]$*? Give your answer in set notation.*

An equivalence class is a list of all the things related to the representative.

Since $(a,b)R(c,d)$ iff $a+b=c+d$, $(2,3)$ is related to any ordered pair that adds to 5.

$$\begin{aligned}\text{So, } [(2,3)] &= \{(x,y)\in\mathbb{Z}\times\mathbb{Z} : x+y=5\}\\ &= \{\ldots(-2,7),(-1,6),(0,5),(1,4),(2,3),\ldots\}.\end{aligned}$$

Let $R = \{(a, b) \ : \ a - b = 0\}$ *be a relation on* $\mathbb{Z}$. *Write a proof that* R *is transitive.*

Theorem: R is transitive.
Theorem: If xRy and yRz, then xRz.

Proof:
Suppose xRy and yRz.
By definition $x - y = 0$ and $y - z = 0$.
Adding z to both sides of $y - z = 0$ gives us $y = z$.
Substituting $y = z$ into $x - y = 0$ gives us $x - z = 0$.
By definition, xRz.

Let $R = \{(a, b) \ : \ b = ak \text{ for some integer } k\}$ *be a relation on integers. R is not an equivalence relation. Why not? Give a counterexample.*

This is saying aRb if b is a multiple of a: $1R2$, $2R6$, etc.

Reflexive: xRx always since $x = x \cdot 1$.

Not symmetric: $1R2$ because 2 is a multiple of 1, but not $2R1$ because 1 is not a multiple of 2.

Let $R = \{(a,b) : b = ak \text{ for some integer } k\}$ *be a relation on integers. Prove that R is transitive.*

Theorem: R is transitive.
Theorem: If xRy and yRz, then xRz.

Proof:
Suppose xRy and yRz.
By definition $y = xj$ and $z = yk$ for some $j, k \in \mathbb{Z}$.
Substituting for y we get $z = (xj)k = x(jk)$.
Because jk is an integer, $z = x\ell$ for some integer $\ell = jk$.
By definition, xRz.