Turing Machine Example

Write a TM that begins with a number in unary on the tape and doubles it.

Before:

111
^  

After:

111111
^
Start with pseudocode

Algorithm design always comes before coding.

Think in terms of simple things a TM can do, like:
- Move right to next blank
- Replace 1 with 0
- Erase 1
- etc.

Each takes a few TM instructions.
Double unary number

High level idea:
- delete a 1 at source
- write two 1s at destination
- repeat until source empty
Double unary number

Low-level pseudocode:

Erase 1
Move right to next blank (end of source)
Move right to next blank (end of destination)
Write two 1s
Move left to next blank (end of source)
Move left
If blank
  Move right twice and exit (Done! Start of destination)
Else
  Move left to next blank
  Move right (to start of source) and goto beginning
Double unary number simulation

11
^ Erase 1

B1
^ Move right to next blank (end of source)

B1B
^
Double unary number simulation

B1B
  ^

Move right to next blank (end of destination)

B1BB
  ^

Write two 1s

B1B11
  ^
Double unary number simulation

B1B11
  ^

Move left to next blank (end of source)

B1B11
  ^

Move left

B1B11
  ^
Double unary number simulation

B1B11
^  

If blank
    Move right twice and exit (Done! Start of destination)
Else
    Move left to next blank
    Move right (to start of source) and goto beginning

B1B11
^  
B1B11
^
Double unary number simulation

B1B11
  ^

Erase 1

BBB11
  ^

Move right to next blank (end of source)

BBB11
  ^
Double unary number simulation

BBB11
   ^

Move right to next blank (end of destination)

BBB11B
   ^

Write two 1s

BBB1111
   ^
Double unary number simulation

BBB1111
   ^

Move left to next blank (end of source)

BBB1111
   ^

Move left

BBB1111
   ^
Double unary number simulation

BBB1111

^  

If blank
   Move right twice and exit (Done! Start of destination)
Else
   Move left to next blank
   Move right (to start of source) and goto beginning

BBB1111

^
Double unary number TM

Turning each step into TM instructions:
Double unary number TM

Each arrow expressed as a TM instruction:

q0 1 B q1
q2 B R q3
q4 B 1 q5
q7 1 L q8
q9 1 L q10
q9 B R q11

q1 B R q2
q3 1 R q3
q5 1 R q6
q8 1 L q8
q10 1 L q10
q11 B R q12

q2 1 R q2
q3 1 R q3
q6 B 1 q7
q8 B L q9
q10 B R q0

If I ask you to give a text representation of a TM, do it this way.
Test

Be sure to give your TM some test inputs to ensure it works correctly.

— Edge cases: zero 1's and one 1.
— General case: two 1's.
Zero 1's

\[ B^\wedge \]

No instruction for this configuration.

Termination.

\[ 0 \times 2 = 0 \]

Correct
One 1

1
^

Erase 1

B
^

Move right to next blank (end of source)

BB
^
Double unary number simulation

BB
^

Move right to next blank (end of destination)

BBB
^

Write two 1s

BB11
^
Double unary number simulation

BB11
  ^

Move left to next blank (end of source)

BB11
  ^

Move left

BB11
  ^
Double unary number simulation

BB11
^ If blank
   Move right twice and exit (Done! Start of destination)
Else
   Move left to next blank
   Move right (to start of source) and goto beginning

BB11
^